TI-Nspire™ CAS Assisted Solutions

VCE Specialist Mathematics Written Sample Examination 2 Section B

Explanatory notes:

Note that the VCAA only supplies multiple-choice answers to sample papers. Every effort has been made to ensure that these solutions are correct.

The author of these solutions has no affiliation with the VCAA.

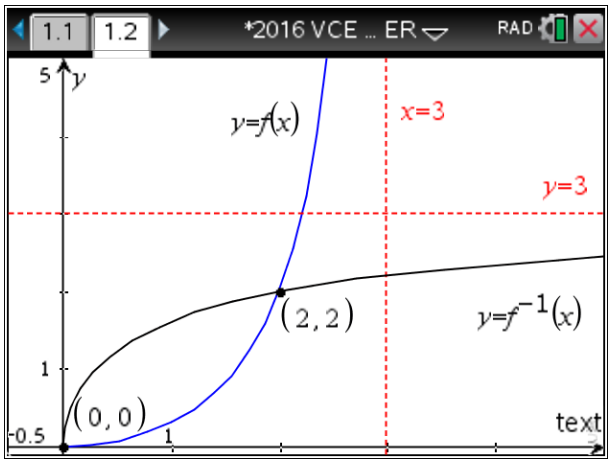
SECTION B – Extended response questions

Question 1:

Part (a)

Working	**TI-Nspire CAS screenshot(s)**
$f(2)=2$	$f(x):=-2+2\cdot\sec\left(\frac{\pi\cdot x}{6}\right)\mid 0\le x<3$ Done $f(2)$ 2

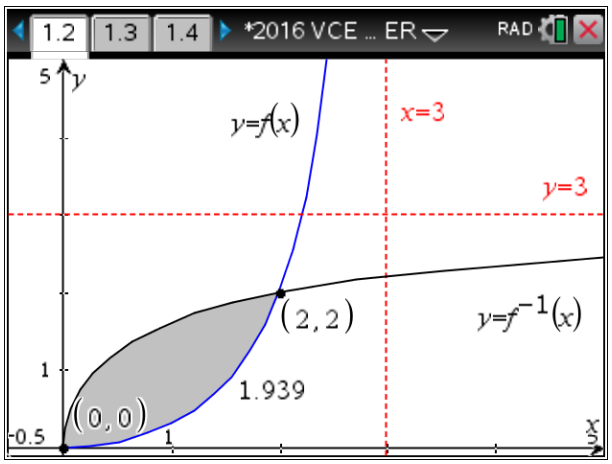
Part (b)

Working	**TI-Nspire CAS screenshot(s)**
f^{-1} is the reflection of f in the line $y=x$ f and f^{-1} intersect at $(0,0)$ and $(2,2)$. On TI-Nspire CAS, graph $x=f(y)$ using the relation graphing feature.	

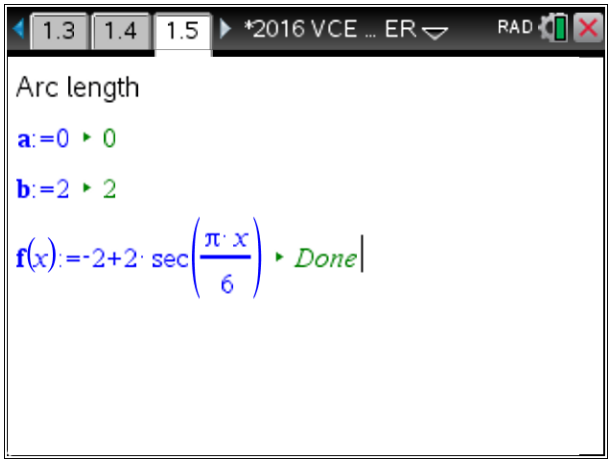
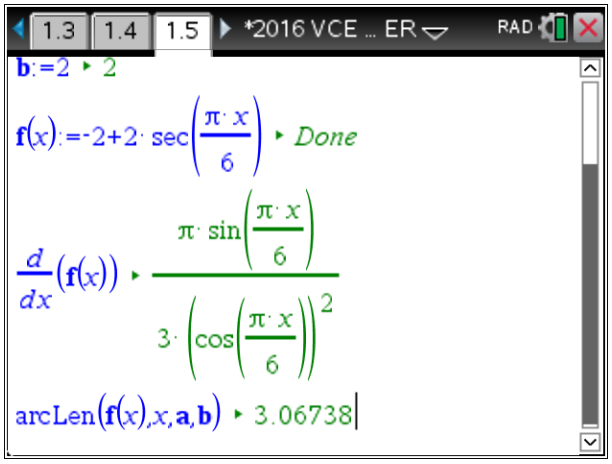
Part (c)

Working	**TI-Nspire CAS screenshot(s)**
Solving $k\arccos\left(\frac{2}{2+2}\right)=2$ for k gives $k=\frac{6}{\pi}$.	$\text{solve}\left(k\cdot\cos^{-1}\left(\frac{2}{2+2}\right)=2,k\right)$ $k=\frac{6}{\pi}$

Part (d)

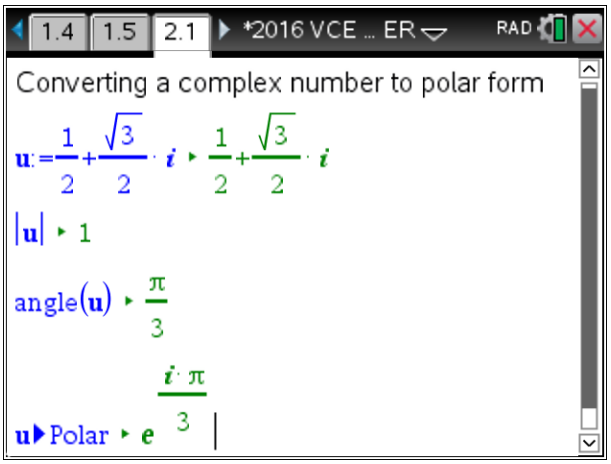
Working	**TI-Nspire CAS screenshot(s)**
$A=\int_0^2\left(f^{-1}(x)-f(x)\right)dx$ So $A=1.939$ (correct to 3 dp).	 $\int_0^2\left(\frac{6}{\pi}\cdot\cos^{-1}\left(\frac{2}{x+2}\right)-f(x)\right)dx$ 1.93918

Parts (e) (i) and (ii)

Working	TI-Nspire CAS screenshot(s)
$L=\int_0^2 \sqrt{1+\left(f'(x)\right)^2}\,dx$ where $f'(x)=\dfrac{\pi\sin\left(\frac{\pi x}{6}\right)}{3\cos^2\left(\frac{\pi x}{6}\right)}$ So $L=3.067$ (correct to 3 dp). The Notes application can be used to calculate the arc length of a curve.	 

Question 2:

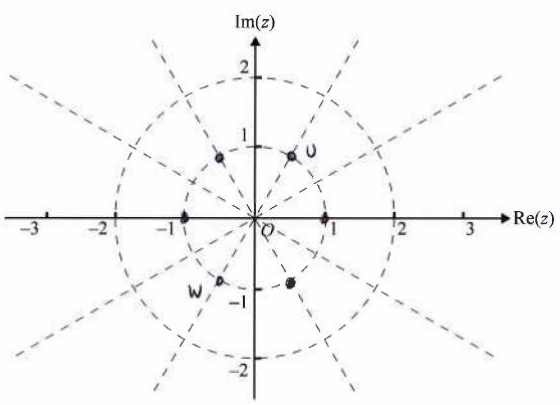
Part (a) (i)

Working	TI-Nspire CAS screenshot(s)
$u=\text{cis}\left(\dfrac{\pi}{3}\right)$ The Notes application can be used to perform complex number arithmetic.	

Part (a) (ii)

Working	**TI-Nspire CAS screenshot(s)**
$u^6 = \text{cis}\left(6 \times \frac{\pi}{3}\right)$ $= \text{cis}(2\pi)$. $= 1$	This question part (a one mark show that question) is best done without the use of TI-Nspire CAS.

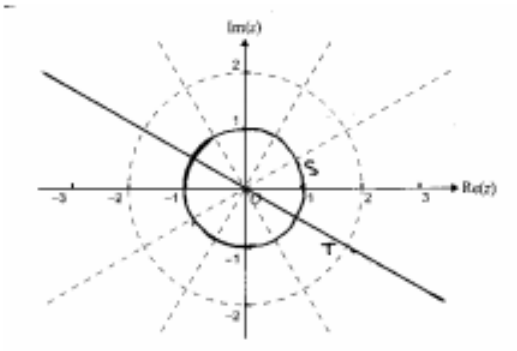
Part (a) (iii)

Working	**TI-Nspire CAS screenshot(s)**
We require 6 points located at: $z = \pm 1, \frac{1}{2} \pm \frac{\sqrt{3}}{2} i, -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$ And we require that u and w are labelled correctly.	

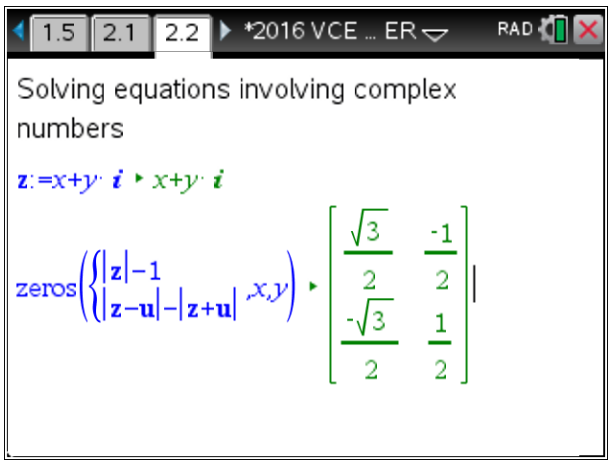
Part (b) (i)

Working	**TI-Nspire CAS screenshot(s)**
Circle centre $(0,0)$ and radius 1.	Diagram is below in (b) (ii).

Part (b) (ii)

Working	**TI-Nspire CAS screenshot(s)**
Straight line passing through $(0,0)$.	

Part (b) (iii)

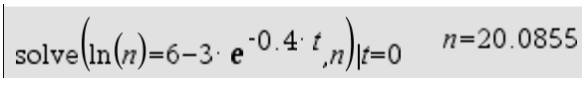
Working	**TI-Nspire CAS screenshot(s)**
By reading the answers from the Argand diagram we obtain $\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$ and $\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$. Alternatively (but not as efficiently): Solving $\lvert z\rvert=1$ and $\lvert z-u\rvert=\lvert z+u\rvert$ for x and y gives the above intersection points.	

Question 3:

Part (a)

Working	**TI-Nspire CAS screenshot(s)**
$\log_e(N)=6-3e^{-0.4t}$ $\frac{d}{dN}\left(\log_e(N)\right)\frac{dN}{dt}=\frac{1}{N}\frac{dN}{dt}$ and $\frac{d}{dt}\left(6-3e^{-0.4t}\right)=1.2e^{-0.4t}$ So $\frac{1}{N}\frac{dN}{dt}=1.2e^{-0.4t}$. Substituting into the LHS of the differential equation gives: $1.2e^{-0.4t}+0.4\left(6-3e^{-0.4t}\right)-2.4=0$	This question part is best attempted without using CAS.

Part (b)

Working	**TI-Nspire CAS screenshot(s)**
$N=20$ (correct to the nearest integer)	

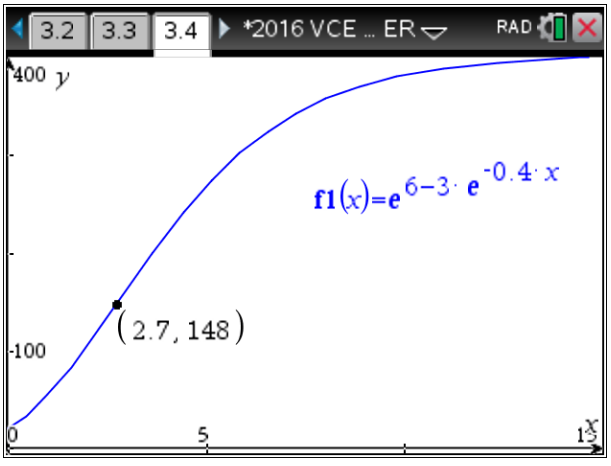
Part (c)

Working	**TI-Nspire CAS screenshot(s)**
When $t \to \infty$, $\log_e(N) \to 6$ and so $N \to 403$ (correct to the nearest integer).	$\lim_{t \to \infty}\left(e^{6-3\cdot e^{-0.4\cdot t}}\right)$ 403.429

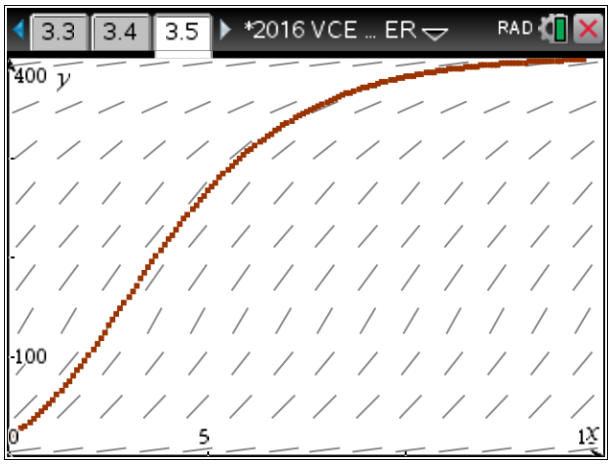
Part (d) (i)

Working	**TI-Nspire CAS screenshot(s)**
$\frac{d^2N}{dt^2} = \frac{d}{dN}\left(\frac{dN}{dt}\right)\frac{dN}{dt}$ $= \left(0.4\left(6-\log_e(N)\right)+0.4N\times\frac{-1}{N}\right)\frac{dN}{dt}$ $= 0.16N\left(5-\log_e(N)\right)\left(6-\log_e(N)\right)$	This question part is best attempted without using CAS.

Part (d) (ii)

Working	**TI-Nspire CAS screenshot(s)**
Solving $\frac{d^2N}{dt^2}=0$ for N gives $N=148$ (correct to the nearest integer). This occurs at $t=2.7$ (years) (correct to 1 dp).	$\text{solve}(5-\ln(n)=0,n)$ $n=148.413$ $\text{solve}\left(\ln(n)=6-3\cdot e^{-0.4\cdot t},t\right)\|n=148.41315910$ $t=2.74653$ 

Part (e)

Working	**TI-Nspire CAS screenshot(s)**
TI-Nspire CAS can be used to help graph the function. See the above right screenshot. Note that the differential equation graphing feature can graph the solution to the differential equation $\frac{dN}{dt} = 0.4N\left(6 - \log_e\left(N\right)\right)$.	

Question 4:

Part (a)

Working	**TI-Nspire CAS screenshot(s)**
$\dot{\underset{\sim}{r}}(0) = 12\cos(60°)\,\underset{\sim}{i} + 12\sin(60°)\,\underset{\sim}{j}$ $= 6\underset{\sim}{i} + 6\sqrt{3}\underset{\sim}{j}$	$[12\cdot\cos(60°) \quad 12\cdot\sin(60°)] \qquad [6 \quad 6\cdot\sqrt{3}]$

Part (b)

Working

$$\underset{\sim}{\dot{r}}(t) = -\frac{t^2}{20}\underset{\sim}{i} - \left(gt - \frac{t^2}{20}\right)\underset{\sim}{j} + \underset{\sim}{c}$$

$\underset{\sim}{\dot{r}}(0) = 6\underset{\sim}{i} + 6\sqrt{3}\underset{\sim}{j}$ and so

$\underset{\sim}{c} = 6\underset{\sim}{i} + 6\sqrt{3}\underset{\sim}{j}$

$$\underset{\sim}{\dot{r}}(t) = \left(6 - \frac{t^2}{20}\right)\underset{\sim}{i} + \left(6\sqrt{3} - gt + \frac{t^2}{20}\right)\underset{\sim}{j}$$

$$\underset{\sim}{r}(t) = \left(6t - \frac{t^3}{60}\right)\underset{\sim}{i}$$

$$+\left(6\sqrt{3}t - \frac{gt^2}{2} + \frac{t^3}{60}\right)\underset{\sim}{j} + \underset{\sim}{d}$$

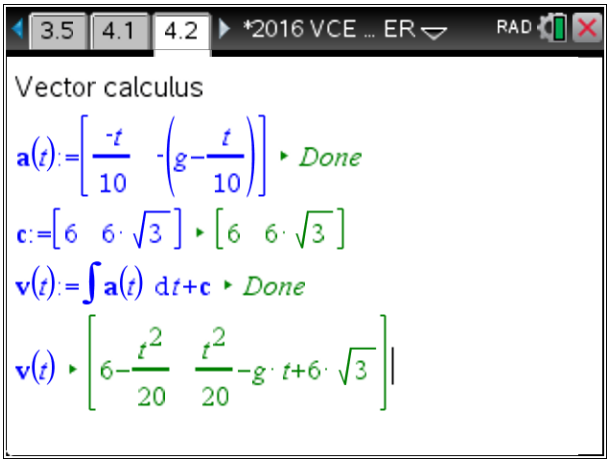
$\underset{\sim}{r}(0) = 0$ and so $\underset{\sim}{d} = \underset{\sim}{0}$

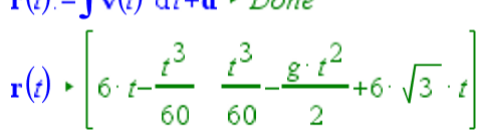
$$\underset{\sim}{r}(t) = \left(6t - \frac{t^3}{60}\right)\underset{\sim}{i}$$

$$+\left(6\sqrt{3}t - \frac{gt^2}{2} + \frac{t^3}{60}\right)\underset{\sim}{j}$$

The Notes application can be used to perform vector calculus calculations.

TI-Nspire CAS screenshot(s)





Part (c)

Working

At $t = T$, the skier lands on the down-slope represented by the equation $y = -x$.

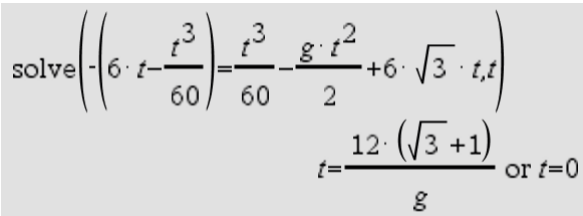
Solving

$$-\left(6T - \frac{T^3}{60}\right) = 6\sqrt{3}T - \frac{gT^2}{2} + \frac{T^3}{60}$$

for T with $T > 0$ gives

$$T = \frac{12}{g}\left(\sqrt{3}+1\right).$$

TI-Nspire CAS screenshot(s)



Part (d)

Working	**TI-Nspire CAS screenshot(s)**
$\left\|\dot{\underset{\sim}{r}}\left(\frac{12}{g}\left(\sqrt{3}+1\right)\right)\right\| = 22.5$ (m/s) (correct to 1 dp) The Notes application can be used to perform vector calculus calculations.	$\text{norm}(\mathbf{v}(t))\|t=\frac{12\cdot(\sqrt{3}+1)}{g}$ and $g=9.8$ ▸ 22.5004

Question 5:

Part (a)

Working	**TI-Nspire CAS screenshot(s)**
Solving $3g - T_1 = 3a$ and $T_1 - g = a$ for a and T_1 gives $a = \frac{g}{2}$ (ms^{-2}). The Notes application can be used to solve equations of motion.	Equations of motion $\text{solve}\left(\begin{cases}3\cdot g-t1=3\cdot a\\ t1-g=a\end{cases},a,t1\right)$ ▸ $a=\frac{g}{2}$ and $t1=\frac{3\cdot g}{2}$

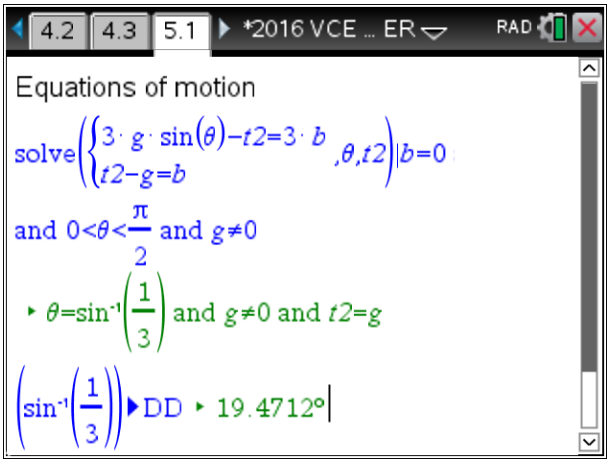
Part (b)

Working	**TI-Nspire CAS screenshot(s)**
Solving the system of equations in (a) gives $T_1 = \frac{3g}{2}$ (N).	See the above screenshot.

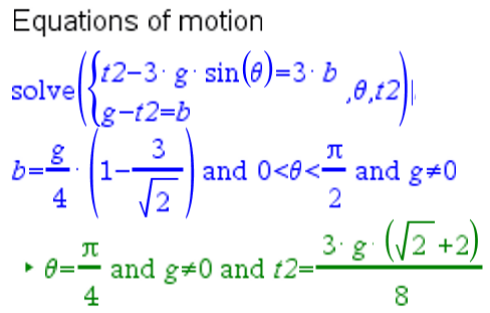
Part (c)

Working	**TI-Nspire CAS screenshot(s)**
Solving $3g\sin(30°) - T_2 = 3b$ and $T_2 - g = b$ for b (and T_2) gives $b = \frac{g}{8}$ (ms^{-2}).	Equations of motion $\text{solve}\left(\begin{cases}3\cdot g\cdot\sin(30°)-t2=3\cdot b\\ t2-g=b\end{cases},b,t2\right)$ ▸ $b=\frac{g}{8}$ and $t2=\frac{9\cdot g}{8}$

Part (d)

Working	**TI-Nspire CAS screenshot(s)**
$b=0$ so $T_2=g$ Solving $3g\sin(\theta)-g=0$ for θ gives $\theta=19.5°$ (correct to 1 dp).	

Part (e)

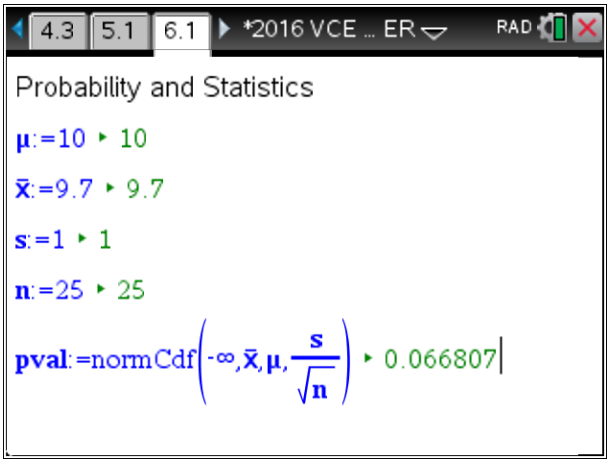
Working	**TI-Nspire CAS screenshot(s)**
Solving $T_2-3g\sin(\theta)=\frac{3g}{4}\left(1-\frac{3}{\sqrt{2}}\right)$ and $g-T_2=\frac{g}{4}\left(1-\frac{3}{\sqrt{2}}\right)$ for θ (and T_2) gives $\theta=45°$.	

Question 6:

Part (a)

Working	**TI-Nspire CAS screenshot(s)**
$H_0:\mu=10$, $H_1:\mu<10$	TI-Nspire CAS functionality does not offer any assistance here.

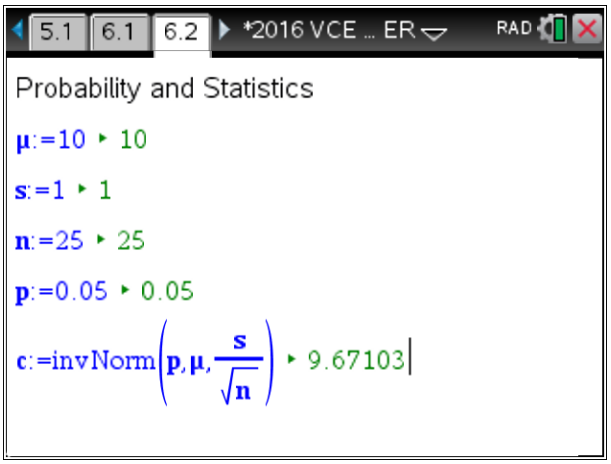
Part (b)

Working	**TI-Nspire CAS screenshot(s)**
$p = \Pr\left(\bar{X} \le 9.7 \mid \mu = 10\right)$ $= 0.067$ (correct to 3 dp) The Notes application can be used to solve probability and statistics exam questions.	

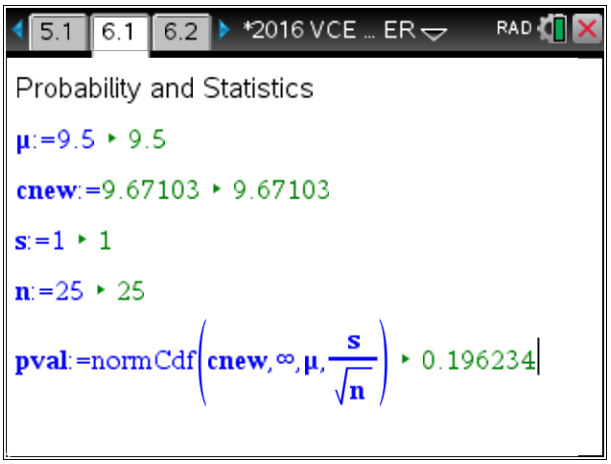
Part (c)

Working	**TI-Nspire CAS screenshot(s)**
Since $p > 0.05(\alpha)$, H_0 is not rejected at the 5% level of significance.	TI-Nspire CAS functionality does not offer any assistance here.

Part (d)

Working	**TI-Nspire CAS screenshot(s)**
Using the inverse normal feature with $p = 0.05$, $\mu = 10$ and $s = \frac{1}{5}$ we obtain $C^* = 9.671$ (correct to 3 dp).	

Part (e) (i)

Working	**TI-Nspire CAS screenshot(s)**
$\Pr\left(\bar{X} > 9.67103 \mid \mu = 9.5\right) = 0.196$ (correct to 3 dp).	

Part (e) (ii)

Working	**TI-Nspire CAS screenshot(s)**
This represents a type II error as it is the same as not rejecting H_0 when it is false. In other words, $\bar{X} > 9.671$ results in H_0 not being rejected even though $\mu = 9.5$.	TI-Nspire CAS functionality does not offer any assistance here.